

Solutions

Exam 2 Chapters 1,2 and 3

Answer the following questions. You must show your work to receive full credit. Be sure to make reasonable simplifications. Give exact answers. Indicate your final answer with a box.

Find the Domain of The Function

1. (4 points) $f(x) = \frac{1}{\sqrt{x-3}}$

Denominator not 0 so $x \neq 3$

$$\sqrt{x-3} \geq 0 \quad \text{so} \quad x \geq 3$$

Thus $\text{dom } f = \boxed{(3, \infty)}$

2. (4 points) Consider the function given by

$$g(w) = 3(w-1)^2.$$

(a) What is the name of the function?

a) g

(b) What letter represents the input?

b) w

(c) What is the output?

(d) Find $g(3)$. What does it represent?

c) $g(w)$ or $3(w-1)^2$

d) $g(3) = 12$, this is the value of g when $w=3$.

3. Your idiot professor is trying to make his dog fly. He discovers that the average helium balloon will hold 4 liters of helium at room temperature and 1 liter of helium will displace 1 gram.

(a) (6 points) Find a function B which models the number of balloons required to lift x grams.

(b) (2 points) How many balloons are required to lift 250 grams?

(c) (2 points) Assuming that it takes 4000 balloons to lift Reggie, how much does he weigh?

1 balloon displaces 4 grams.

$$(a) \quad B(x) = \frac{1}{4}x$$

$$(b) \quad B(250) = \frac{1}{4} \cdot 250 = 62.5 \text{ balloons}$$

$$(c) \quad 4000 = B(x) = \frac{1}{4}x \Rightarrow x = 16,000 \text{ grams}$$

4. (4 points) A score of years is 20 years. When Abraham Lincoln said "4 score and 7 years ago," how long ago was he referring to?

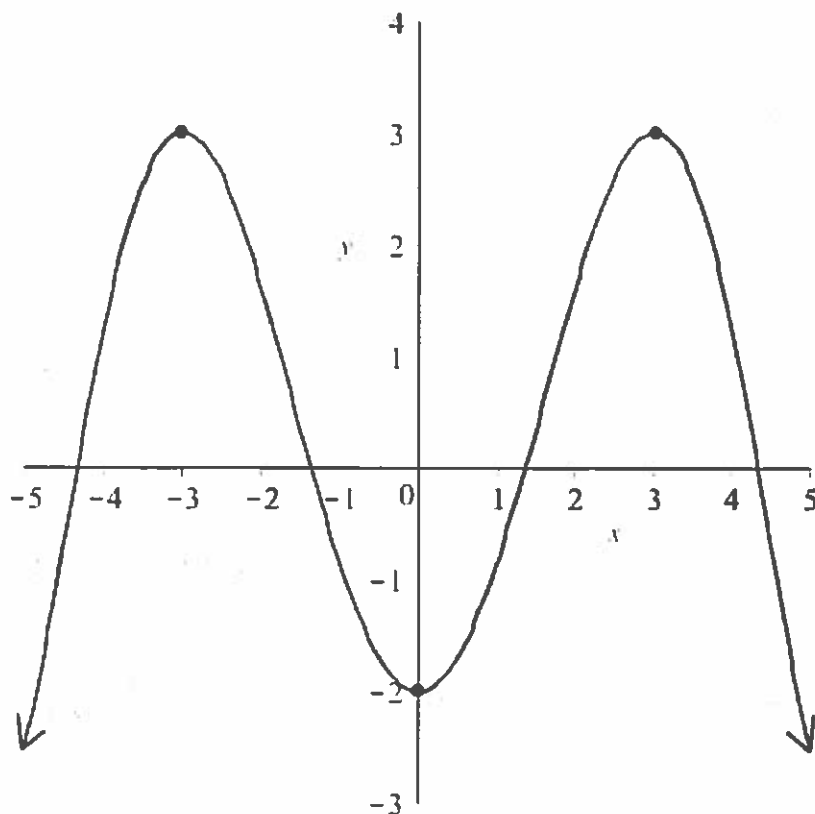
87 years

5. (4 points) Find a function that models the number N of days in w weeks.

$$N = 7w$$

6. (2 points) A linear function has constant rate of change.

7. (9 points) A graph of a function g is given below.



- (a) Find the intervals on which g is increasing and the intervals on which g is decreasing. Express your answer in interval notation.
- (b) Find all local minimum/maximum values of g and the values of x at which they occur.
- (c) Find the domain and range of g . Express your answer in interval notation.

(a) Increasing $(-\infty, -3] \cup [0, 3]$

Decreasing $[-3, 0] \cup [3, \infty)$

(b) local max = 3 occurs at $x = \pm 3$
local min = -2 occurs at $x = 0$

(c) $\text{Dom}(g) = (-\infty, \infty)$ $\text{Ran}(g) = (-\infty, 3]$

8. (9 points) Consider the three lines given below.

Line A is given by $y = x + 2$
Line B is given by $y = -2x + 11$
Line C is given by $y = x - 1$

For each pair of lines below determine the point of intersection. If there is no intersection, state clearly the reason.

(a) Lines A and B

$$(a) \quad x + 2 = -2x + 11 \Rightarrow 3x = 9 \Rightarrow x = 3$$

(b) Lines B and C

$$y = 3 + 2 = 5 \quad \text{intersect at } (3, 5)$$

(c) Lines A and C

$$(b) \quad -2x + 11 = x - 1 \Rightarrow 3x = 12 \Rightarrow x = 4$$

$$y = 4 - 1 = 3 \quad \text{intersect at } (4, 3)$$

(c) Parallel so no intersection.

9. (4 points) Let l be the line given by the equation $y = 3x + 1000$.

(a) Find the line parallel to l passing through the point $(1, 6)$.

$$(a) \quad y - 6 = 3(x - 1)$$

(b) Find the line perpendicular to l passing through the point $(3, -2)$.

$$(b) \quad y + 2 = -\frac{1}{3}(x - 3)$$

10. On October 17, 2008, the average price of a gallon of gas in California was \$3.60, which was a drop of 10% from the previous week. Forecasters predicted that the trend would continue and the price of gas would be under \$2.00 by the end of November. Assume the price of gas decreased at the forecasted decay rate each week.

- (a) (5 points) Find the weekly decay factor, a , and find an exponential growth model $G(x)$ for the price of gas x weeks after October 17, 2008, when the price was \$3.60 per gallon.
- (b) (2 points) Use the model to predict the price of gas at the end of November (6 weeks later). Is this what the forecasters predicted?

(a) Decay Factor = 0.9

$$G(x) = 3.60(0.9)^x$$

(b) $G(6) = 3.60(0.9)^6 \approx \1.91 per gallon.

Yes their prediction follows the model.

11. The bacteria *R. sphaeroides* is the cause of the disease chlamydia. A culture of these bacteria initially has 25 bacteria and increases at a rate of 28% per hour.

- (a) (5 points) Find an exponential growth model for the bacteria count in the sample, where t is measured in hours.
- (b) (2 points) Use the model you found to predict the number of bacteria in the sample after 7 hours.

$$(a) P(t) = 25 \cdot 1.28^t$$

$$(b) P(7) = 25 \cdot 1.28^7 \approx 140.74 \text{ bacterium}$$

12. The bacterium *Brucella melatensis* is one of the causes of mastitis infections in milking cows. A lab tests the growth rates of two strains of this bacterium:

Strain A increases by 40% every 4 hours

Strain B increases by 30% every 2 hours

- (a) (6 points) Find the one-hour growth factor for each strain.
- (b) (2 points) Which strain has the larger growth rate?

$$(a) A: 4 \text{ hour factor} = 1.4 \Rightarrow 1 \text{ hr factor} = \sqrt[4]{1.4} \approx 1.088$$

$$B: 2 \text{ hour factor} = 1.3 \Rightarrow 1 \text{ hr factor} = \sqrt{1.3} \approx 1.140$$

(b) B has larger growth rate

13. (4 points) Determine if the following table can be represented by an exponential model, linear model or neither. Explain why. If so, find the model.

x	$f(x)$
0	3000
1	2440
2	1880
3	1320
4	760

Handwritten annotations: On the left, four vertical arrows point downwards from the x-values 0, 1, 2, and 3. On the right, four vertical arrows point downwards from the f(x) values 3000, 2440, 1880, and 1320, with the number 560 written next to each arrow.

Linear

$$f(x) = -560x + 3000$$

BONUS QUESTION What are you going to be for Halloween?

Ben Wyatt

